Exercise 97

The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

Solution

Write formulas for the volume and surface area of a cube in terms of its edge length x.

$$\begin{cases} V = x^3 \\ \\ S = 6x^2 \end{cases}$$

In order to get the rates at which these quantities change as time increases, differentiate both sides of each equation with respect to t and use the chain rule.

$$\begin{cases} \dfrac{d}{dt}(V) = \dfrac{d}{dt}(x^3) \\ \\ \dfrac{d}{dt}(S) = \dfrac{d}{dt}(6x^2) \end{cases}$$

$$\begin{cases} \dfrac{dV}{dt} = 3x^2 \dfrac{dx}{dt} \\ \\ \dfrac{dS}{dt} = 12x \dfrac{dx}{dt} \end{cases}$$

Now set $dV/dt = 10$ and $x = 30$.

$$\begin{cases} 10 = 3(30)^2 \dfrac{dx}{dt} \\ \\ \dfrac{dS}{dt} = 12(30) \dfrac{dx}{dt} \end{cases}$$

This is a system of two equations for dx/dt and dS/dt. Solve the first equation for dx/dt,

$$\frac{dx}{dt} = \frac{10}{3(30)^2} = \frac{1}{270},$$

and plug it into the second equation.

$$\frac{dS}{dt} = 12(30)\left(\frac{1}{270}\right) = \frac{4}{3}$$

Therefore, the surface area is increasing at a rate of about 1.333 cm^2/min when the edge length is 30 cm.